Distributed Work Stealing for Constraint Solving
(Extended Abstract)

Vasco Pedro and Salvador Abreu

Departamento de Informática, Universidade de Évora and
CENTRIA FCT/UNL, Portugal
{vp,spa}@di.uevora.pt

Abstract. With the dissemination of affordable parallel and distributed hardware, parallel and distributed constraint solving has lately been the focus of some attention. To effectually apply the power of distributed computational systems, there must be an effective sharing of the work involved in the search for a solution to a Constraint Satisfaction Problem (CSP) between all the participating agents, and it must happen dynamically, since it is hard to predict the effort associated with the exploration of some part of the search space. We describe and provide an experimental assessment of an implementation of a work stealing-based approach to parallel CSP solving in a distributed setting.

1 Introduction

Constraints are used to model problems with no known polynomial algorithm, but for which search techniques developed within the field of constraint programming provide viable procedures. Besides classical applications, such as planning and scheduling, constraints have recently been successfully applied in the contexts of bioinformatics and computer network monitoring [11, 12].

Notwithstanding their relative efficiency, constraint solving methods are computationally demanding and good candidates to benefit from multiprocessing. Moreover, the declarative style of constraint programming frees the programmer from concerns usually entailed by parallel and distributed programming, such as control, synchronisation, and communication issues. In fact, the programmer may not even be aware that there is any parallelism involved in solving the problem. Given the increasing availability of parallel computational resources, in the form of multiprocessors, clusters of computers, or both, there is a need for an effective way to help incorporating that power into the constraint programming setting.

Constraint solving involves exploring large search spaces. To perform search using several agents in parallel, the search effort must be shared among them. In *distributed constraint solving*, in the context of solving Distributed CSPs [17], each agent does a part of the work and coordinates with the other agents in order to find a solution. The present work follows the *parallel constraint solving* approach [4, 15, 13, 7, 2], where the search space is partitioned and the search for a solution is carried out in each of the sub-search spaces by one agent (or worker), all agents working in parallel. Here the agents are mostly independent from each other, performing their (non-overlapping) part of the work and hoping that one of them will find a quicker path to an answer. While the first approach typically requires significant inter-agent communication, not only for the search to progress but also for termination detection, in the latter communication can be limited to an initial dispatching of the agents and to an answer collecting phase at the end of the procedure. In this case, however, the initial work distribution may turn out to be quite unbalanced, leaving some agents to bear most of the effort as others become idle and their contribution is wasted.

This article reports on preliminary results of our experiments in implementing a work-stealing scheme for overcoming the effect described above. This is a two-level scheme: work stealing occurs between co-located agents, but when distant agents are involved, some cooperation is needed to redistribute the work still left.

The remainder of this paper is structured as follows: we start by establishing some terminology in the next section. Then, in Sections 3 and 4 we describe the architecture of the implemented solver and report on some experimental results obtained with it. Section 5 discusses related work and in Section 6 we conclude and put forward possible continuation paths for this work.

2 Constraint Solving

A constraint satisfaction problem can be briefly defined as a set of variables whose values, to be drawn from their domains, must satisfy a set of relations.

Definition 1 (CSP). *A* Constraint Satisfaction Problem *(CSP) over finite domains is a triple $P = (X, D, C)$, where*

- $X = \{x_1, x_2, \ldots, x_n\}$ *is an indexed set of* variables*;*
- $D = \{D_1, D_2, \ldots, D_n\}$ *is an indexed set of finite sets of values, with D_i being the* domain *of variable x_i, for every $i = 1, 2, \ldots, n$; and*
- $C = \{c_1, c_2, \ldots, c_m\}$ *is a set of relations between variables, called the* constraints*.*

The *search space* of a CSP consists of all the tuples from the cross product of the domains, where each variable is assigned a value from its domain. Solving a CSP amounts to finding some or all of those tuples which satisfy all constraints of the problem.

Definition 2 (Solution). *A* solution *to a CSP is an n-tuple $(v_1, v_2, \ldots, v_n) \in D_1 \times D_2 \times \ldots \times D_n$ such that all constraints are satisfied.*

In parallel constraint solving, the problem is divided into subproblems. Solutions to these subproblems are also solutions to the original problem.

Definition 3 (Subproblem). *A* subproblem *of a CSP $P = (X, D, C)$ is a CSP $P' = (X, D', C)$ such that $D' = \{D'_1, D'_2, \ldots, D'_n\}$ and $D'_i \subseteq D_i$, for every $i = 1, 2, \ldots, n$.*

To guarantee completeness of the search, the search spaces of the subproblems must cover the search space of the original problem. In order to avoid redundant work, they must also be pairwise disjoint.

Definition 4 (Partition). *A set $\{P'_1, P'_2, \ldots, P'_k\}$ of subproblems of a CSP P, with $P'_i = (X, \{D'_{i1}, D'_{i2}, \ldots, D'_{in}\}, C)$, is a* partition *of P if*

$$\bigcup_{1 \le i \le k} D'_{i1} \times D'_{i2} \times \cdots \times D'_{in} = D_1 \times D_2 \times \cdots \times D_n$$

and $(\forall\, i \neq j)\, D'_{i1} \times D'_{i2} \times \cdots \times D'_{in} \cap D'_{j1} \times D'_{j2} \times \cdots \times D'_{jn} = \emptyset$.

A partition of a CSP may be dually regarded as a partition of its search space, the search spaces of the subproblems being sub-search spaces of the original problem. In this paper we will only deal with search space partitions that correspond to some partition of a problem.

3 Solver Architecture

Our constraint solver consists of *workers*, grouped together as *teams* (Figure 1). The search for one or all solutions is carried out by the workers, which implement a propagator based constraint solving engine, following a domain consistency oriented approach [1]. Each active worker has a *pool* of *idle* search spaces and a *current* search space, the one it is currently exploring. In each team there is a *controller*, which does not participate in the search, and one of the controllers, the *main controller*, also coordinates the teams.

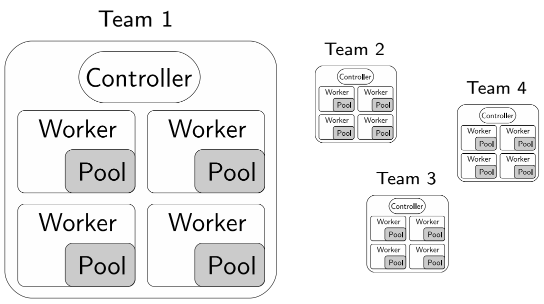


Fig. 1. Solver architecture

Structuring the workers this way serves two purposes: the first is that a workers' sole task becomes searching, as all communication with the environment required by the dynamic sharing of work among teams is handled by the controller. The second objective is the sharing of resources enabled by binding the workers in a team close together. If all workers were on the same level, they would either have to divide their attention between search and communication or there would have to be one controller per worker, thereby increasing resource usage. On the other hand, this structure matches naturally a two-level partitioning of the search space and we obtain receiver-initiated decentralised dynamic load balancing [16].

At the outset of the search process, the problem to be solved is partitioned and each team is entrusted with trying to solve one of the resulting subproblems. The controller in each team then partitions the local problem and hands each sub-search space over to a worker for exploration.

On finishing exploring its assigned search space, a worker tries to steal work from another worker *within its team*. If unsuccessful, it then notifies the team controller that it has become idle. When all the workers in a team are idle, the controller asks the other teams for more work.

3.1 Partitioning Strategies

The strategy used to partition the search space has a decisive impact on the number of steps needed to get to a solution, hence on performance.

Partitioning strategies may be designed either to lead to a balanced distribution of the search work, like the *even* strategy below and the prime and greedy strategies from [14], or to produce some subproblems where the search is expected to be quick (while others may be slow), such as *eager* partitioning. In principle, the former strategies will be more suited to situations where all solutions are requested and the whole search space must be visited, and the latter will lend themselves better to when one solution is enough. In any case, the splitting of the problem will introduce a breadth-first component into the usual depth-first exploration of the search tree, which sometimes gives rise to superlinear speedups.

In *even partitioning*, domains are split so as to obtain sub-search spaces of similar dimensions. If we want to split a problem into k subproblems, then the first variable with at least that many values in its domain is chosen and its domain is split as evenly as possible among the subproblems: if the domain of the chosen variable has $d \geq k$ values, then it will have $\lfloor d/k \rfloor$ values in the first $k - d \,\mathbf{mod}\, k$ subproblems and $\lfloor d/k \rfloor + 1$ values in the remaining $d \,\mathbf{mod}\, k$ subproblems.

Eager partitioning corresponds roughly to a partial breadth-first expansion of the search tree and it will mostly produce subproblems where at least one of the variables has had its domain reduced to a single value. The splitting is performed according to the algorithm depicted in Figure 2, whose inputs are the number of subproblems to create and a sequence of problems from which to create them. Initially, this sequence only contains the original problem.

The partitioning of the CSP may affect the behaviour of the search, even to the point of defeating the variable and value selection heuristics which are usually appropriate to a given problem, as has been noted in [7, Section 6]. This

Notation *If P is a CSP and D is a finite set, PD_i stands for the CSP which is identical to P except that the domain of the i^{th} variable is D.*

```
eager-split($k, (P_1\, P_2 \cdots P_q)$)
    $(X, D, C) \leftarrow P_1$
    $i \leftarrow \min \{j \mid |D_j| > 1\}$
    $d \leftarrow |D_i|$
    $\{v_1, v_2, \ldots, v_d\} \leftarrow D_i$
    if $k \leq d$ then
        $(P_1\{v_1\}_i\; P_1\{v_2\}_i \cdots P_1\{v_k, \ldots, v_d\}_i\; P_2 \cdots P_q)$
    else
        eager-split($k - d + 1, (P_2 \cdots P_q\; P_1\{v_1\}_i\; P_1\{v_2\}_i \cdots P_1\{v_d\}_i)$)
```

Fig. 2. Eager partitioning algorithm

suggests that the partitioning strategy, introducing another degree of freedom in the search strategy, needs to be adapted to the problem being solved and matched with the search heuristics used, and that no overall 'best' partitioning strategy exists. (Notice that, for the present, problem specific heuristics do not inform problem partitioning.)

As problem partitioning takes place at two points in the process — to distribute work to all the teams, and, initially within every team, to assign work to each worker — different splitting strategies can be used, a more balanced one to allot similar amounts of work to the individual teams, and another to focus the efforts of the agents. The latter strategy could be finer grained than the former, the cost of local work stealing being much lower than that of network supported work sharing.

3.2 Search

The search unfolds as a worker further splits the search space it is working on, keeping one part as its current search space and adding the other to its pool of idle search spaces. If the current search space is found to contain no solution, the worker draws a new search space from the pool and starts exploring it, never backtracking. Upon finding a solution, the worker communicates it to the team controller which, in turn, forwards it to the main controller.

The state of a worker with two search spaces currently in the pool is shown in Figure 3, where solid edges mean that the child search spaces form a partition of the parent. Notice that the subtree to the left of the current search space (corresponding to the tuples where both x_1 and x_2 take value 1) has already been explored and discarded, and is not displayed.

Figure 4 depicts the main driver algorithm for workers. At each step of the search process, a worker starts by looking within its current search space for a variable whose domain is not a singleton (line 3). If none is found, then the search space contains a single tuple which constitutes a solution to the problem,

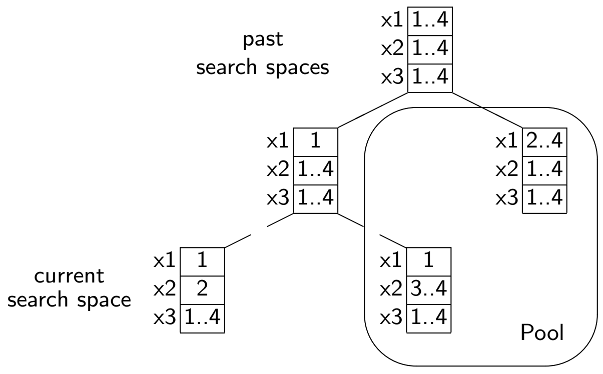


Fig. 3. Search spaces from a worker

and which is returned by the worker (line 10). Otherwise, one of the variables with a non-singleton domain is selected and the current search space is split into two subspaces (line 4):

- In the first, which will become the worker's current search space, the selected variable is set to an individual value picked from its domain.
- In the other, to be added to the pool of idle search spaces (line 5), that value is removed from the domain of the variable.

The domains of the other variables remain unchanged in both search spaces.

Following the split, the new current search space goes through a propagation phase (line 6). If it succeeds, another search step is performed. If the propagation fails, the worker tries to fetch an idle search space from the pool to become the current search space (line 7). If this is not possible the worker fails (line 9), otherwise the search resumes with the retrieved search space undergoing a propagation phase, as the domain of one of its variables shrunk just prior to it being stored in the idle pool.

```
 1: WORKER(search-space)
 2:   current ← search-space
 3:   while var ← select-variable(current) do
 4:     (current, other) ← split-search-space(var, current)
 5:     pool-put(other, var)
 6:     while (current ← revise(var, current)) = FAIL do
 7:       (current, var) ← pool-get()
 8:       if current = FAIL then
 9:         return FAIL
10:   return SOLUTION(current)
```

Fig. 4. Worker main driver algorithm

3.3 Work Stealing

When a worker tries to fetch a new search space from its pool and finds it empty, it will attempt to obtain one from one of its teammates. In order to minimise the impact on the performance of the solver, this is achieved with as little cooperation from the holder of the retrieved search space as possible. In fact, the idle worker will effectively steal work from a teammate while the latter continues its task, oblivious to what is being done to its work queue.

The intended discipline of a worker's pool is that of a *deque* (double-ended queue), as depicted in Figure 5. While the owner works on one end of its pool (lines 2, 8, and 12), a worker whose pool is empty will remove an entry from the other end (line 20). This way, the only penalty a worker incurs during normal processing is the cost of an extra check on the size of its pool (line 6). The protocol used to avoid interference during pool accesses is similar to the one in [5]. Only when the number of entries in the pool is small, will it be necessary to enforce mutual exclusion in the accesses to the pool, and even then only when removing a search space. To reduce contention, work stealing is only allowed from a pool when the number of entries in it reaches a given threshold (line 17).

```
 1: pool-put(search-space, variable)
 2:     pool.append(search-space, variable)

 3: pool-get()
 4:     if pool.size = 0 then
 5:         return steal-work()
 6:     else if pool.size < SAFE-SIZE then
 7:         lock(pool)
 8:         ss ← pool.remove-last()
 9:         unlock(pool)
10:         return ss
11:     else
12:         return pool.remove-last()

13: steal-work()
14:     lock(stealing)
15:     v ← worker-with-biggest-pool()
16:     lock(v.pool)
17:     if v.pool.size < THRESHOLD then
18:         ss ← FAIL
19:     else
20:         ss ← v.pool.remove-first()
21:     unlock(v.pool)
22:     unlock(stealing)
23:     return ss
```

Fig. 5. Pool insertion and removal, and work stealing algorithm

Stolen work corresponds to locations nearer the root of a worker's search tree. The search within the worker's search space proceeds according to the heuristics deemed adequate to the problem until it either finds a solution or the work is exhausted. Upon stealing work from a peer, a worker picks up the search at a point that the worker it was stolen from would eventually reach, thus subverting the problem's search strategy and introducing in it a measure of randomness. This may be either beneficial or detrimental, depending on the specific problem.

In the event of an idle worker failing to obtain work within its team, it notifies the team controller and waits, either to be later restarted or to be terminated. When all the agents in a team have become idle, the team controller broadcasts a request for more work to the other teams.

Inter-team work stealing follows along a simple plan: initially, one of the team controllers is given the role of fulfilling requests for work. Upon receiving one, and using the same protocol used by the workers, it tries to steal a search space from the local pool to be forwarded to the requester, which splits it among its workers and becomes the new work supplier. If the designated work supplier is unable to spare a search space, the remaining teams are polled for work, as done in [13]. When no team is able to supply additional work, the idle team notifies the main controller and terminates.

3.4 Implementation Notes

One of the main goals behind this work was to build a constraint solver which could take advantage of the advances in parallel architectures and in clustering network technology. To better be able to handle the challenges inherent to multiprocessing, namely memory management and caching issues, C was our choice for the implementation language, as it allows for very fine-grained control.

Teams are autonomous entities and each team corresponds to a distinct process, usually residing on a dedicated machine. As communication, particularly over a network, may have an adverse impact on system performance, care has been taken to minimise the number of inter-team messages needed. Teams are coordinated by way of an IPC library.

A team comprises active components which are the workers and the controller. The controller is, most of the time, waiting for a worker or another team controller to communicate with it, not disturbing the search process and allowing workers to be mapped to processors. Workers are mostly independent from each other, except where work stealing is concerned, as explained in Section 3.3. A worker, to be able to steal work from another one without active cooperation from the latter, must be able to access all the team pools. To make this possible, pools are located in shared memory and workers, as well as the controller, are implemented as lightweight processes (threads).

4 Experimental Results

In this Section, we present some performance results obtained with our solver on two classic benchmark problems, namely the non attacking queens problem and the Langford number problem [3, problem 024]. While the queens problem has many solutions well spread out throughout the search space, the Langford number problem either has no solution or it also has many solutions but not so well distributed.

Measurements were made of the time taken to count all solutions for the two problems and for generating the first solution in the second problem. These measurements were made on a cluster of Q6600 Intel Core2 Quad CPUs, clocked at 2.4GHz, with 2–4GB RAM, running Linux, and the code was compiled with GCC 4.1.1 with the '-O3' flag. The times presented are the average of 12 runs of each program, with the worst and the best times excluded. When computing the relative performance with respect to the sequential case, we subtracted the overhead associated with starting up and terminating the solver, which reached a maximum of 0.3 seconds in the 6 teams configuration. Unless otherwise indicated, teams are composed of 4 workers, mirroring the number of CPUs in the shared-memory multiprocessor systems. For interprocess communication, the Open MPI MPI-2 implementation [9] was used.

Absolute performance has not, so far, been the top priority goal of this work. Nevertheless the sequential (1 team with 1 worker) version of our solver already displays interesting times for solving these problems, as attested by Table 1, where they are compared with those of Gecode 3.0.2 [6], although there clearly remains some work to be done in that regard.

Table 1. Times comparison with Gecode (seconds)

	Queens			Langford				
	14	15	16	2 11	2 12	2 28	2 31	3 18
Our solver	13.89	86.05	580.36	1.07	8.00	67.56	1.26	2.44
Gecode	17.21	102.18	646.43	36.40	25.01	0.03	0.02	0.42
	all solutions					first solution		

In the remainder of this section, we look at the results obtained with several configurations of the solver and analyse them with respect to the speedups induced by the parallelisation of the search, using the two partitioning strategies. The use of the two strategies helps both to illustrate the consequences of problem partitioning and to highlight the effect of work stealing.

In the non attacking queens problem, the first observation that can be made in relation to the speedups obtained, depicted in Figure 6, is that they are fairly insensitive to the partitioning strategy used. Given that in this problem the work is very evenly distributed among the possible values from the domains of the variables, this result is only possible due to effective work sharing.

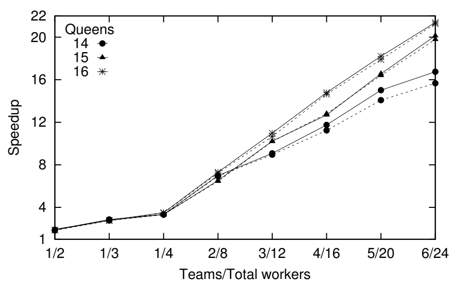
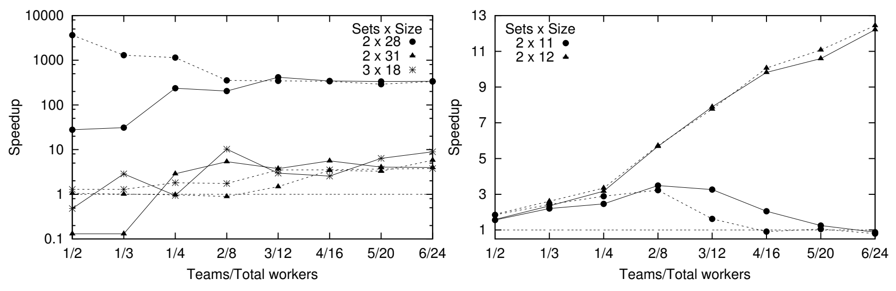


Fig. 6. Speedups for the non attacking queens (all solutions)[1]

[1] In these graphs, solid and dashed lines correspond, respectively, to even and eager partitioning.

The profile of the speedups evolution with the addition of more teams is quasi-linear for the 16 queens problem, showing good scalability of the approach. However, the smaller problem starts suffering from the weight of the implementation early on. Total running times for the three problems in the 6 team setting are around 1.2, 4.6, and 27.5 seconds, for 14, 15, and 16 queens, respectively.

The Langford number problem, for which we measured both the speedups for counting all solutions and for obtaining the first solution, is an example of a case where domain partitioning interacts badly with the heuristics usually used for guiding the search, as dividing a domain gives rise to more work than that needed to solve the original problem. This is apparent in Figure 7a, which represents the results observed in finding the first solution and where some instances of the problem displayed a marked slowdown when partitioning the domain of the first variable in two or three similarly sized parts. On the other hand, speedups of more than 3000 were also obtained in one case.



(a) First solution **(b)** All solutions

Fig. 7. Speedups for the Langford number problem

Counting all solutions of the Langford problem (Figure 7b) exhibits a profile common to the previous problem, but at some point the implementation starts overwhelming the potential improvements due to the parallelisation on the smaller instance. This effect requires further study to identify and solve its causes.

5 Related Work

Recent years have seen an increase in the interest in parallel solving, as parallel architectures become more common. An early language sporting parallel constraint solving was the CHIP parallel constraint logic programming language [15]. It was implemented on top of the logic programming system PEPSys, whose or-parallel resolution infrastructure was adapted to handle the domain operations needed in parallel constraint solving.

More recent works rely on features of an underlying framework for programming parallel search. The concurrent Oz language provides the basis for the implementation described in [13], where search is encapsulated into computation spaces and a distributed implementation allows the distribution of workers. Work sharing is coordinated by a manager, which receives requests for work from the workers and then tries to find one willing to share the work it has left. Search strategies are user programmed and the work sharing strategy is implemented by the workers.

A similar approach is taken in [7, 8] which show how to program parallel search controllers in Comet. There, the pool is an active object which is queried by the idle workers. In case the pool is empty, it asks another worker to generate yet unexplored sub-search spaces, gives one away and stores the rest. It is not explained, however, how the worker which supplies work is chosen.

A focus of research has been on the strategies for splitting the work between workers. These strategies may be driven by the problem structure, such as the size of the domains [14], or by the past behaviour of the solver, be it related with properties of the solving process, such as the number of variables already instantiated [10], or with the progress of the search, in what it affects the prospects of finding a solution in the current subtree [18] or in the subtrees left to explore [2]. Here, a scheme is presented which uses the search heuristics to guide problem splitting, dampened by a degree of confidence to distribute the workers across the search tree while maintaining some bias towards the nodes favoured by the heuristic. It shows good performance on multi-core hardware, and while it has the drawback of working on a global view of the search process, it seems to point in a promising direction of research, namely using the work done as a guide to future search space splitting.

6 Conclusions and Future Work

In spite of the results obtained so far, there should be additional gains with a more sophisticated work sharing protocol. Several possibilities should be studied, including having a different work stealing policy for inter-team sharing, where candidate search spaces undergo a deeper examination to try to determine whether the cost of their sending is offset by the work saved locally.

Short term development plans comprise improving the internal representation of the domains, which currently only allows values between 0 and 63, the inclusion of optimisation constraints, and the improvement of the scalability of the implementation in two key aspects: the initial work distribution and the sharing of work between teams, which could both profit from organising the teams in some way.

We also plan on experimenting with different underlying models and libraries for thread management and inter-process communication, namely to venture beyond the present implementation which relies on Posix threads and MPI.

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